

LIMITS

Math 130 - Essentials of Calculus

12 February 2021

USING CONTINUITY TO EVALUATE A LIMIT

EXAMPLE

Consider the function $f(x) = \frac{x^2 + x - 6}{x - 2}$.

- 1 What is the domain of f ?

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Consider the function $f(x) = \frac{x^2 + x - 6}{x - 2}$.

- 1 What is the domain of f ?
- 2 Compute $\lim_{x \rightarrow 4} f(x)$.
- 3 Compute $\lim_{x \rightarrow 2} f(x)$.

NOW YOU TRY IT!

EXAMPLE

Consider the function $f(x) = \frac{x^2 - 2x - 3}{x + 1}$.

- 1 What is the domain of f ?
- 2 Compute $\lim_{x \rightarrow 1} f(x)$.
- 3 Compute $\lim_{x \rightarrow -1} f(x)$.

MORE LIMITS

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$$\textcircled{1} \lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7}$$

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$$\textcircled{2} \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$$

$$\textcircled{3} \lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4+x}$$

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Compute the limits

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$$\textcircled{2} \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$$

$$\textcircled{3} \lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4+x}$$

$$\textcircled{4} \lim_{x \rightarrow 0} \frac{1}{x^2}$$

ONE-SIDED LIMITS

EXAMPLE

Consider the function

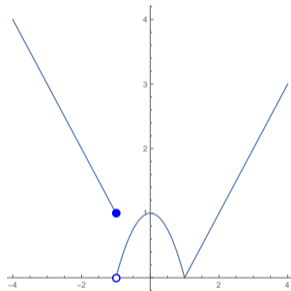
$$g(x) = \begin{cases} -x, & x \leq -1 \\ 1 - x^2, & -1 < x < 1 \\ x - 1, & x \geq 1 \end{cases}$$

Compute the following limits:

① $\lim_{x \rightarrow 1^+} g(x)$

② $\lim_{x \rightarrow 1} g(x)$

③ $\lim_{x \rightarrow 0} g(x)$



④ $\lim_{x \rightarrow -1^-} g(x)$

⑤ $\lim_{x \rightarrow -1^+} g(x)$

⑥ $\lim_{x \rightarrow -1} g(x)$

RELATION BETWEEN ONE-SIDED AND TWO-SIDED LIMITS

THEOREM

$$\lim_{x \rightarrow a} f(x) = L \quad \text{if and only if} \quad \lim_{x \rightarrow a^-} f(x) = L \text{ and } \lim_{x \rightarrow a^+} f(x) = L$$